Multinomial Logistic Regression (MLR):
(aka polytomous-response)

- Choice when dependent variable is nominal and > 2 categories (when = 2, use binary logistic regression)
- If not sure whether categories are ordered, use MLR
- Requires fewer or weaker assumptions than ordinal logit

Model

$$\text{Prob}(y = j) = \frac{e^{\sum_{k=1}^{K} \beta_{jk} x_k}}{1 + \sum_{j=1}^{J-1} e^{\sum_{k=1}^{K} \beta_{jk} x_k}}$$

- Where j=1, 2 …. J-1
- β Parameters have two subscripts: k for distinguishing x vars and j for distinguishing response categories
- There are J-1 sets of β estimates [total # of parameters = (J-1)K]

Binary dependent variable, simplifies to:

$$\text{Prob}(y=1) = \frac{e^{\sum_{k=1}^{K} \beta_k x_k}}{1 + e^{\sum_{k=1}^{K} \beta_k x_k}}$$

Identical to :

$$P(y=1) = \frac{e^{XB}}{1+e^{XB}}$$

Probability for reference category

$$P(y=J) = \frac{1}{1+\sum_{j=1}^{J-1} e^{\sum_{k=1}^{K} \beta_{jk} x_k}}$$

or,

$$P(y=J) = 1 - [\text{Prob}(y=1) + \ldots \text{Prob}(y=J-1)]$$

Multinomial model in logit form

$$\log\left[\frac{\text{Prob}\ (y = j)}{\text{Prob}\ (y = J)}\right] = \sum_{k=1}^{K} \beta_{jk} x_k$$

When J = 2, simplifies to :

$$\log\left[\frac{\text{Prob}\ (y = 1)}{1 - \text{Prob}\ (y = 1)}\right] = \sum_{k=1}^{K} \beta_k x_k$$

IIa Assumption

- Important methodological point: *independence from irrelevant alternatives*
- IIa property holds that the ratio of choice probabilities of any two alternatives is not influenced systematically by any other alternative
- Example: red bus/blue bus paradox

Source: Liao, 1994, p. 50