

Multinomial Logistic Regression (MLR): (aka polytomous-response)

- Choice when dependent variable is nominal and > 2 categories (when = 2, use binary logistic regression)
- If not sure whether categories are ordered, use MLR
- Requires fewer or weaker assumptions than ordinal logit

Model

$$\text{Prob}(y = j) = \frac{e^{\sum_{k=1}^K \beta_{jk} x_k}}{1 + \sum_{j=1}^{J-1} e^{\sum_{k=1}^K \beta_{jk} x_k}}$$

- Where $j=1, 2, \dots, J-1$
- β Parameters have two subscripts: k for distinguishing x vars and j for distinguishing response categories
- There are $J-1$ sets of β estimates [total # of parameters = $(J-1)K$]

Binary dependent variable,
simplifies to:

$$\text{Prob}(y = 1) = \frac{e^{\sum_{k=1}^K \beta_k x_k}}{1 + e^{\sum_{k=1}^K \beta_k x_k}}$$

Identical to :

$$P(y = 1) = \frac{e^{XB}}{1 + e^{XB}}$$

Probability for reference category

$$P(y = J) = \frac{1}{1 + \sum_{j=1}^{J-1} e^{\sum_{k=1}^K \beta_{jk} x_k}}$$

or,

$$P(y = J) = 1 - [\text{Prob}(y = 1) + \dots + \text{Prob}(y = J - 1)]$$

Multinomial model in logit form

$$\log \left[\frac{\text{Prob} (y = j)}{\text{Prob} (y = J)} \right] = \sum_{k=1}^K \beta_{jk} x_k$$

When $J = 2$, simplifies to :

$$\log \left[\frac{\text{Prob} (y = 1)}{1 - \text{Prob} (y = 1)} \right] = \sum_{k=1}^K \beta_k x_k$$

IIA Assumption

- Important methodological point:
independence from irrelevant alternatives
- IIA property holds that the ratio of choice probabilities of any two alternatives is not influenced systematically by any other alternative
- Example: red bus/blue bus paradox

Source: Liao, 1994, p. 50

Interpretation: MLR

(positive coefficients; odds ratios > 1)

- Logit coefficients: being black increases the log odds of being in masculine occupations relative to whites
- Odds ratios: Blacks had over two times the odds of being in masculine occupations as whites

Interpretation: MLR

(negative coefficients; odds ratios < 1)

- Logit coefficients: being a citizen reduces the log odds of being in masculine occupations relative to whites
- Odds ratios: the odds for citizens to be in masculine occupations are only .678 times as high as those for noncitizens